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Pseudo-scalar heavy meson decays in covariant light front dynamics

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Abstract. We investigate the leptonic decay amplitude of heavy pseudo-scalar mesons as well as the B to D transition amplitude in the heavy quark limit, using the covariant formulation of light-front dynamics. The explicit rotational invariance of this formalism enables us to recover the analytic results obtained for these processes using the Bakamjian–Thomas construction in equal time dynamics, in the limit of an infinitely heavy mass. The covariant formulation provides a very convenient and coherent framework for calculating the $1/m$ corrections.

1 Introduction

The leptonic decay and semi-leptonic transition form factors of heavy mesons are a subject of particular interest for several reasons. In the framework of the standard model, they are directly proportional to matrix elements of the CKM matrix, and therefore can serve as constraints on their determination. This, however, implies that the uncertainties coming from the calculation of the hadronic transition amplitudes can be minimized as much as possible. This is indeed the case if one considers heavy mesons containing c or b quarks or antiquarks. In that case, and in leading order in $1/m$, where m is the mass of the heavy quark, several properties arise from the so-called heavy quark symmetry [1,2].

Such a symmetry, which implies various sum rules, can be used for instance to constrain the slope of the Isgur–Wise function $\xi(w)$ as $w \to 1$ [3–7]. It is now well known that such constraints can only be satisfied if the relevant matrix elements are described covariantly. In other words, one needs a coherent relativistic framework to describe the initial and final state, as well as the electroweak operator.

Several attempts have been made in the past to satisfy these constraints at least in $1/m$ order. Among them, the construction of covariant amplitudes using the Bakamjian–Thomas (BT) construction is the most widely used [8,9]. This construction however may have limitations in estimating non-leading $1/m$ corrections. We would like to advocate in this article the use of light front dynamics (LFD) to calculate in a systematic way the semi-leptonic decay of the heavy mesons. Some results are already known in the standard formulation of LFD [3,10–12,7,13,14]. While the numerical results, in leading $1/m$ order, in both the LFD and the BT approaches are identical, an explicit comparison beween their results is not always possible because of the lack of explicit covariance (i.e. also the lack of rotational invariance) in the standard formulation of LFD.

Moreover, and this is the aim of the present study, the lack of rotational invariance may lead to large non-physical contributions to any approximate physical amplitude, as this is already known for electromagnetic form factors [15–18]. In order to extract unambiguously the physical form factors, we therefore use in the present work the covariant formulation of LFD (CLFD), as detailed in the review article of [19]. We apply our analysis to the leptonic decay of $B(0^-)$ mesons, and to the semi-leptonic transition $B(0^-)$ to $D(0^-)$.

The plan of this paper is as follows. We recall in Sect. 2 the general structure of the pseudo-scalar meson wave function in CLFD. We calculate in Sect. 3 the leptonic meson decay constant, and in Sect. 4 the semi-leptonic transition form factor between two heavy mesons. We draw our conclusions in Sect. 5.

2 Relativistic structure of scalar mesons in CLFD

2.1 Covariant formulation of LFD

Among the various approaches to deal with relativity in the description of bound states, we shall concentrate in the following on light front dynamics. In the standard formulation of LFD, the wave function of the system is defined

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on a plane characterized by the equation $t + z/c = 0$. The usual Schrödinger, equal time, formalism is easily recovered by letting c go to infinity.

The formulation of relativistic systems in LFD has many advantages. Maybe the most important one is the absence of vacuum fluctuations. This has the important consequence that a meaningful decomposition of the state vector describing the system under consideration in terms of Fock components of a definite number of particles is possible. The number of Fock components to be considered in any practical calculation depends of course on the dynamics of the system, and on the kinematical regime one is interested in.

The most serious drawback of this formulation is however that the position of the light front $t + z = 0$ (with $c = 1$) is not invariant by any rotation in the zx and zy planes. Since these rotations change the position of the light front, the associated generators should depend on the dynamics and cannot be reduced to kinematical transformations [20]. This means in practice that one needs to know the complete dynamics in order to write down the general structure of a bound state of definite angular momentum. This means also that any electroweak operator should have the same (dynamical) transformation properties in order to match those of the bound state wave function. This is essential in order to guarantee that any physical amplitude (or cross-section) does not depend on the particular choice of the light front we start with.

One therefore needs an explicit procedure to exhibit in a convenient way these dynamical transformations. This can be achieved easily in the covariant formulation of LFD. Our starting point is the invariant definition of the light front by $\omega.x = 0$, where ω is an (unspecified) light-like four-vector ($\omega^2 = 0$). If one specifies a particular value of ω, i.e. for instance $\omega = (1, 0, 0, -1)$, one recovers the standard formulation of LFD.

This definition of the light front is explicitly invariant by any four-dimensional rotation, or any three-dimensional rotation and Lorentz boost. As a consequence, these transformations become entirely kinematical, but ω-dependent, and do not necessitate the knowledge of the dynamics of the system to construct them explicitly. All the dynamics is now described by the ω-dependence of the wave function and the electroweak operator, in such a way that any physical amplitude should not depend on the particular position of the light front, i.e. should not depend on ω, unless approximations have been made. In that case – which is almost always the case in practice – the explicit covariance of the approach enables us to exhibit the ω-dependence of the amplitude and to extract the physical part from the non-physical, ω-dependent one, as we shall explain in the following in the case of the decay amplitude and transition form factors. All the details of this covariant formulation can be found in [19].

2.2 The two-body wave function

Following the spirit of the constituent quark model, we approximate the heavy meson wave function by its two-body Fock component only. The state vector describing a meson, of momentum p and light front "time " $\sigma = \omega{\cdot}x$, on the light front defined by ω, is thus given by [19]

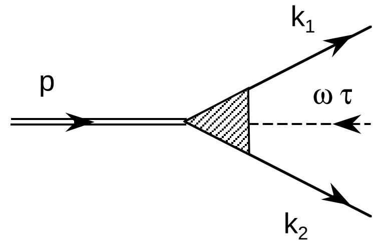

Fig. 1. Graphical representation of the two-body wave function on the light front characterized by ω. The broken line corresponds to the spurion (see text)

$$\begin{aligned}\phi^{J\lambda}(p) &= (2\pi)^{3/2} \int \Phi^{J\lambda}_{j_1\sigma_1 j_2\sigma_2}(k_1, k_2, p, \omega\tau) \\ &\times a^\dagger_{\sigma_1}(\vec{k}_1) a^\dagger_{\sigma_2}(\vec{k}_2)|0\rangle \delta^{(4)}(k_1 + k_2 - p - \omega\tau) \\ &\times \exp(\mathrm{i}\tau\sigma) 2(\omega{\cdot}p)\mathrm{d}\tau \frac{\mathrm{d}^3 k_1}{(2\pi)^{3/2}\sqrt{2\varepsilon_{k_1}}} \frac{\mathrm{d}^3 k_2}{(2\pi)^{3/2}\sqrt{2\varepsilon_{k_2}}}, \quad (1)\end{aligned}$$

in an obvious notation, and $\varepsilon_k = (\vec{k}^2 + m^2)^{1/2}$. Note that the four-momentum conservation can be written $\delta^{(4)}(k_1 + k_2 - p - \omega\tau)$. This originates from the general transformation properties of the state vector with respect to the generators of the Poincaré group [19]. In the particular case where $\omega = (1, 0, 0, -1)$, the delta-function $\delta^{(4)}(k_1 + k_2 - p - \omega\tau)$ gives, after integration over τ, the standard conservation laws for the $(\perp, +)$-components of the momenta, but does not constrain the minus-components. For convenience, it is useful to represent this wave function by the diagram of Fig. 1. The dashed line, called a spurion, of momentum $\omega\tau$, is just a systematic way to take care of the particular four-momentum conservation. It enters also naturally in the diagrammatic rules associated with CLFD, which generalize the Weinberg rules [19].

From (1) one can see that the wave function depends on the orientation of the light front, from its argument $\omega\tau$, where τ is homogeneous to an energy. This latter is entirely fixed by the four-momentum conservation and the on-mass-shell conditions for each particle. The dependence of any Fock component on ω is very natural. Indeed any off-energy-shell amplitude is related to the S-matrix defined on a finite light front plane in the interaction region and therefore depends on its orientation. The bound state wave function is always an off-shell object ($\tau \neq 0$ due to binding energy). Therefore it also depends on the orientation of the light front plane. This property is not a peculiarity of the covariant approach. The covariance allows one however to parametrize this dependence explicitly.

The general transformation properties of the wave function under a four-dimensional rotation g is given by [19]

$$\Phi^{J\lambda}_{j_1\sigma_1 j_2\sigma_2}(gk_1, gk_2, gp, g\omega\tau) = \sum_{\lambda'\sigma_1'\sigma_2'} D^{(J)*}_{\lambda\lambda'}\{R(g,p)\} D^{(j_1)}_{\sigma_1\sigma_1'}\{R(g,k_1)\} D^{(j_2)}_{\sigma_2\sigma_2'}\{R(g,k_2)\} \times \Phi^{J\lambda'}_{j_1\sigma_1' j_2\sigma_2'}(k_1, k_2, p, \omega\tau). \quad (2)$$

Here $D^{(J)}_{\lambda'\lambda}\{R(g,p)\}$ is the matrix of the rotational group and $R(g,p)$ is the rotation operator:

$$R(g,p) = L^{-1}(gp)gL(p), \quad (3)$$

where $L(p)$ is the Lorentz transformation corresponding to the velocity $\vec{v} = \vec{p}/p_0$. The Euler angles that determine the rotation $R(g,p)$ can be expressed in terms of the momentum p and the parameters of the transformation g. The explicit expression of the Euler angles in terms of g and p will however never be needed.

The wave function Φ can be decomposed in terms of all the possible independent spin structures compatible with the quantum numbers of the meson. In the particular case of pseudo-scalar mesons we are interested in, we can write

$$\Phi = \frac{1}{\sqrt{2}} \bar{u}(k_2) \left[A_1 + A_2 \frac{\not{\omega}}{\omega \cdot p} \right] \gamma_5 v(k_1). \quad (4)$$

As we can see in this expression, the scalar wave function has two components. The first one, A_1, gives rise to the usual Schrödinger wave function in the non-relativistic limit. The second one, A_2, has a relativistic origin and has no equivalence in the non-relativistic approach.

In order to make a close connection to the non-relativistic limit, it is convenient to make use of another set of variables. We denote by $\vec{k}$ the momentum which corresponds, in the c.m. system where $\vec{k}_1 + \vec{k}_2 = 0$, to the usual relative momentum between the two particles. Note that this choice of variable does not assume, however, that we restrict ourselves to this particular reference frame. We denote by $\vec{n}$ the unit vector in the direction of $\vec{\omega}$ in this system. Note also that due to the four-momentum conservation law, the total momentum $\vec{p}$ of the system in this reference frame is not zero. In terms of these variables, the wave function takes a form very similar to the non-relativistic case. Making the appropriate Lorentz transformations, we get

$$\vec{k} = L^{-1}(\mathcal{P})\vec{k}_1 = \vec{k}_1 - \frac{\vec{\mathcal{P}}}{\sqrt{\mathcal{P}^2}} \left[k_{10} - \frac{\vec{k}_1 \cdot \vec{\mathcal{P}}}{\sqrt{\mathcal{P}^2} + \mathcal{P}_0} \right], \quad (5)$$

$$\vec{n} = L^{-1}(\mathcal{P})\vec{\omega}/|L^{-1}(\mathcal{P})\vec{\omega}| = \sqrt{\mathcal{P}^2} L^{-1}(\mathcal{P})\vec{\omega}/\omega\cdot p, \quad (6)$$

where

$$\mathcal{P} = p + \omega\tau = k_1 + k_2, \quad (7)$$

From these definitions, it follows that under a rotation and a Lorentz transformation g of the four-vectors from which $\vec{k}$ and $\vec{n}$ are formed, the vectors $\vec{k}$ and $\vec{n}$ undergo only rotations:

$$\vec{k}' = R(g,\mathcal{P})\vec{k}, \quad \vec{n}' = R(g,\mathcal{P})\vec{n}, \quad (8)$$

where R is the rotation operator (3). Therefore $\vec{k}^2$ and $\vec{n}\cdot\vec{k}$ are invariants.

One can see from (2) that the relativistic wave function, in contrast to the non-relativistic one, is transformed in each index by different rotation matrices. It is therefore convenient to use a representation in which the wave function is transformed in each index by one and the same rotation operator $R(g,\mathcal{P})$, rotating, according to (8), the variables $\vec{k}$ and $\vec{n}$. We define the wave function in this new representation as follows:

$$\Psi^{J\lambda}_{j_1\sigma_1 j_2\sigma_2}(k_1, k_2, p, \omega\tau) \equiv \sum_{\lambda',\sigma_1',\sigma_2'} D^{(J)*}_{\lambda\lambda'}\{R(L^{-1}(\mathcal{P}),p)\} D^{(j_1)}_{\sigma_1\sigma_1'}\{R(L^{-1}(\mathcal{P}),k_1)\} \times D^{(j_2)}_{\sigma_2\sigma_2'}\{R(L^{-1}(\mathcal{P}),k_2)\} \Phi^{J\lambda'}_{j_1\sigma_1' j_2\sigma_2'}(k_1, k_2, p, \omega\tau), \quad (9)$$

where, e.g., $R(L^{-1}(\mathcal{P}),p)$ is given by (3) with $g = L^{-1}(\mathcal{P})$.

The transformation properties of the wave function in the new representation are thus

$$\Psi^{\lambda}_{\sigma_1\sigma_2}(gk_1, gk_2, gp, g\omega\tau) = \sum_{\lambda',\sigma_1'\sigma_2'} D^{(J)*}_{\lambda\lambda'}\{R(g,\mathcal{P})\} D^{(j_1)}_{\sigma_1\sigma_1'}\{R(g,\mathcal{P})\} \times D^{(j_2)}_{\sigma_2\sigma_2'}\{R(g,\mathcal{P})\} \Psi^{\lambda'}_{\sigma_1'\sigma_2'}(k_1, k_2, p, \omega\tau). \quad (10)$$

This equation, together with (8), shows that in this new representation and in the variables $\vec{k}, \vec{n}$ the relativistic wave function transforms exactly as a non-relativistic wave function under a rotation R. This strongly simplifies the spin structure of the relativistic wave function, making it as close as possible to the non-relativistic one. The only difference is in the dependence of the wave function on the extra variable $\vec{n}$.

In terms of these variables, we can write the pseudo-scalar meson wave function as

$$\psi(\vec{k},\vec{n}) = \frac{1}{\sqrt{2}} w_2^{\dagger} \left(g_1 + \mathrm{i}\frac{\vec{\sigma}\cdot(\vec{n}\times\vec{k})}{|\vec{k}|} g_2 \right) w_1, \quad (11)$$

where w_1 and w_2 are the usual two-component Pauli spinors. The two functions g_1 and g_2 are scalar functions of two invariants. It is convenient to choose $\vec{k}^2$ and $\vec{k}.\vec{n}$. The relation between the components in both representations can easily be found using the explicit expression of the Dirac spinors [19]. In the non-relativistic limit, the second component proportional to g_2 disapears, and the wave function g_1 tends to the non-relativistic Schrödinger wave function which we shall denote by $\phi_{NR}(\vec{k}^2)$. It depends on $\vec{k}^2$ only.

The equation for the wave function is shown graphically in Fig. 2. It is the analogue, for a bound state, of the Lippmann–Schwinger equation for a scattering state. For many practical applications, it may be useful to express this equation in terms of the variables $\vec{k}$ and $\vec{n}$, i.e. express the wave function in terms of $\Psi^{\lambda}_{\sigma_1\sigma_2}(k_1, k_2, p, \omega\tau) \equiv$

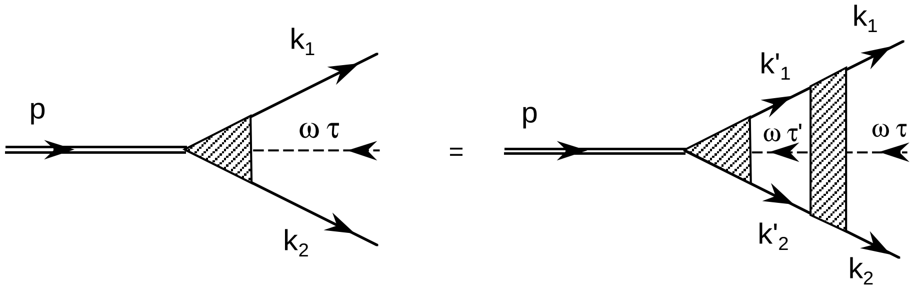

Fig. 2. Diagrammatical representation of the self-consistent equation determining the two-body relativistic wave function

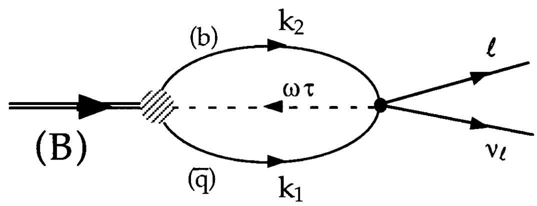

Fig. 3. Leading order contribution to the leptonic decay amplitude of pseudo-scalar heavy mesons

$\Psi^{\lambda}_{\sigma_1\sigma_2}(\vec{k},\vec{n})$. In the simple case of a scalar particle, we have

$$\left[4(\vec{k}^2+m^2)-M^2\right]\psi(\vec{k},\vec{n}) = -\frac{m^2}{2\pi^3}\int \psi(\vec{k}',\vec{n})V(\vec{k}',\vec{k},\vec{n},M^2)\frac{\mathrm{d}^3k'}{\varepsilon_{k'}}. \quad (12)$$

The integration variable $\vec{k}'$ in (12) is defined analogously to $\vec{k}$ in (5):

$$\vec{k}' = L^{-1}(\mathcal{P}')\vec{k}'_1, \quad (13)$$

where $\mathcal{P}' = k'_1 + k'_2$. The kernel V of this equation (shaded area on the right side of Fig. 2) depends on the dynamics of the system.

3 The B meson decay constant

As a first application of CLFD to heavy quark systems, we shall concentrate in this section on the decay constant of heavy pseudo-scalar mesons, like $B(0^-)$. The leading order contribution is indicated in Fig. 3. According to the diagrammatical rules associated with CLFD [19], the spurion line connects two successive vertices in the light front time $\omega{\cdot}x$.

The decay constant is thus deduced from the general structure of the exact current J_ρ by

$$\langle 0|J_\rho|0^-\rangle = f_P p_\rho, \quad (14)$$

where p_ρ is the momentum of the meson. Since we restrict ourselves to the first two-body Fock component of the state vector (1), and evaluate the leading contribution to the current as given in Fig. 3, the hadronic matrix element we calculate is approximate, and can therefore depend a priori on the orientation of the light front on which the wave function is defined. Since our formulation is explicitly covariant, we can immediately write down the structure of our approximate matrix element, which for simplicity we shall also denote by $\langle 0|J_\rho|0^-\rangle$. We have thus

$$\langle 0|J_\rho|0^-\rangle = f_P p_\rho + B\omega_\rho. \quad (15)$$

B is a non-physical contribution which should be disantangled from the physical contribution, f_P, to the exact decay constant. In this case, this can simply be done by multiplying both sides of (15) by ω_ρ so that

$$f_P = \frac{\langle 0|\omega{\cdot}J|0^-\rangle}{\omega{\cdot}p}. \quad (16)$$

With the current given diagrammatically by Fig. 3, we have [19]:

$$\langle 0|J_\rho|0^-\rangle = \frac{\sqrt{N_c}}{\sqrt{2}}\int \frac{\mathrm{d}^3k_1}{(2\pi)^3 2\varepsilon_{k_1}}\frac{1}{1-x} \times \mathrm{Tr}\left[(m_q - \not{k}_1)\left(A_1 + A_2\frac{\not{\omega}}{\omega\cdot p}\right)\gamma_5(m_b+\not{k}_2)\gamma_\rho\gamma_5\right], \quad (17)$$

with $x = \omega{\cdot}k_1/\omega{\cdot}p$, and where N_c is the number of colors. The masses of the heavy quarks are denoted by m_b and m_c, and m_q is the mass of the light quark.

We shall evaluate the decay constant in the infinite quark limit. According to the usual procedure in this limit, we shall make the following approximations:

$$m_b, m_c, M_B, M_D \to \infty \quad \text{and} \quad m_q \ll m_b, m_c, M_B, M_D,$$

$$\text{with} \qquad \frac{m_b}{M_B} \to 1, \qquad \frac{m_c}{M_D} \to 1.$$

From this we can deduce immediately, with $p + \omega\tau = k_1 + k_2$:

$$\tau \approx \frac{m_b^2 - M_B^2}{2\omega\cdot p} \to 0. \quad (18)$$

We thus find that in this limit, the reference frame, where $\vec{p}=0$ (rest frame of the initial meson), is identical to the center of mass frame of the two particles of momentum $\vec{k}_1$ and $\vec{k}_2$ ($\vec{k}_1 + \vec{k}_2 = 0$). We shall denote by $\vec{k}$ the momentum of the light antiquark in this frame, as defined in (5).

In this limit, the fraction of the momentum carried by the light antiquark, and given by x, is thus

$$x = \frac{\varepsilon_k}{m_b}\left(1 - \frac{\vec{n}.\vec{k}}{\varepsilon_k}\right), \quad (19)$$

with $\varepsilon_k = (\vec{k}^2 + m_q^2)^{1/2}$. In order to evaluate the contribution coming from the relativistic component A_2 of the two-body wave function, we can estimate this component in perturbation theory, as done in [21] for the deuteron wave function. Using (12), we start from the non-relativistic wave function Ψ_{NR} on the r.h.s. of the equation, as defined by

$$\psi(\vec{k},\vec{n}) = \frac{1}{\sqrt{2}} w_2^\dagger \left(\phi_{NR}(\vec{k}^2)\right) w_1, \quad (20)$$

and calculate the complete, two-component, wave function $\Psi(\vec{k},\vec{n})$ on the l.h.s., assuming a particular model for V. In the infinite mass limit however, since $\tau \to 0$, the component A_2 of the relativistic wave function also goes to zero. It can therefore be neglected in this limit.

The component A_1 is then calculated by comparing the two decompositions (4) and (11). It is given by

$$A_1(\vec{k}^2, \vec{n}.\vec{k}) = \frac{g_1(\vec{k}^2, \vec{n}.\vec{k})}{\sqrt{2m_b}\sqrt{\varepsilon_k + m_q}}, \quad (21)$$

with the normalization [19]

$$\int \frac{\mathrm{d}^3k}{(2\pi)^3 2\varepsilon_k} |g_1(\vec{k}^2, \vec{n}.\vec{k})|^2 = 1. \quad (22)$$

Note that the normalization incorporates a factor $2\varepsilon_k$ as compared to the standard non-relativistic normalization.

By identifying g_1 with the non-relativistic wave function $\phi_{NR}(\vec{k}^2)$, and with the definition of f_P in (16), with (17), we finally get

$$f_P = \sqrt{N_c}\sqrt{\frac{2}{M_B}} \int \frac{\mathrm{d}^3k}{(2\pi)^3\sqrt{2\varepsilon_k}} \phi_{NR}(\vec{k}^2) \sqrt{\frac{\varepsilon_k + m_q}{\varepsilon_k}}. \quad (23)$$

With the normalization (22) this expression is identical to the one obtained in [9] using the Bakamjian–Thomas construction. Relativistic corrections to the decay constant can indeed be large, even in the infinite mass limit, since they involve $\varepsilon_k = (\vec{k}^2 + m_q^2)^{1/2}$, where m_q is the *light* quark or antiquark mass. They lead to a reduction of f_P. Numerical results using various realistic potentials can be found in [9].

4 The semi-leptonic transition form factors

4.1 Formulation of the amplitude in CLFD

The exact physical amplitude for the semi-leptonic transition form factor $B \to D + l\nu_l$ is in the Fermi approximation usually decomposed as follows:

$$\mathcal{M} = \frac{G_f V_{bc}}{\sqrt{2}} \langle 0^-|J^\rho|0^-\rangle J_\rho^{\mathrm{lep}}, \quad (24)$$

where J_ρ^{lep} is the standard leptonic current.

The hadronic matrix element is further decomposed in terms of the f_+ and f_- form factors, according to [22]

$$\langle 0^-|J^\rho|0^-\rangle = (p+p')^\rho f_+ + (p-p')^\rho f_-. \quad (25)$$

In the heavy quark approximation, a more convenient parametrization is usually considered:

$$\langle 0^-|J^\rho|0^-\rangle = \sqrt{M_B M_D}\left[(U+U')^\rho h_+ + (U-U')^\rho h_-\right], \quad (26)$$

where M_B and M_D are the masses of the initial and final mesons, and U and U' are the four-velocities defined by

$$U = \frac{p}{M_B} \quad \text{and} \quad U' = \frac{p'}{M_D}. \quad (27)$$

In this equation, p and p' are the momenta of the initial and final mesons respectively. This decomposition is the most useful since in the heavy quark approximation the form factor h_- must be zero, so that we have to deal with only one physical form factor. This form factor is a function of $Q^2 = (p-p')^2$. In the heavy quark limit, it is more convenient to express Q^2 in terms of the variable $\eta = U \cdot U'$, with

$$Q^2 = M_B^2 + M_D^2 - 2M_B M_D \eta. \quad (28)$$

The form factor h_+ is normalized to one at the point of zero recoil ($\eta = 1$), and its slope near 1 is the so-called Isgur–Wise function ρ^2:

$$h_+(\eta) = 1 - \rho^2(\eta - 1) + O\left[(\eta-1)^2\right]. \quad (29)$$

As we already mentioned, the exact physical transition amplitude defined on the light front should of course not depend on the particular choice of its orientation, i.e. on ω. But this is not the case in any approximate calculation. The explicit covariance of our approach enables us, however, to parametrize this dependence explicitly. The approximate amplitude is thus given by

$$\langle 0^-|J^\rho|0^-\rangle = \sqrt{M_B M_D}\left[(U+U')^\rho h_+ + (U-U')^\rho h_-\right] + B\omega^\rho, \quad (30)$$

where B is a non-physical form factor which should be zero in any exact calculation. It is now easy to invert this decomposition in order to extract the physical form factors h_+ and h_- from the amplitude. Using the following scalar products, and with $\alpha = \omega \cdot U'/\omega \cdot U$:

$$\begin{cases} \mathcal{X} = (U+U')\cdot J = 2\sqrt{M_B M_D}(1+\eta)h_+ \\ \quad +(1+\alpha)(\omega\cdot U)B, \\ \mathcal{Y} = (U-U')\cdot J = 2\sqrt{M_B M_D}(1-\eta)h_- \\ \quad +(1-\alpha)(\omega\cdot U)B, \\ \mathcal{Z} = \dfrac{\omega\cdot J}{\omega\cdot U} = \sqrt{M_B M_D}\left[(1+\alpha)h_+ + (1-\alpha)h_-\right], \end{cases} \quad (31)$$

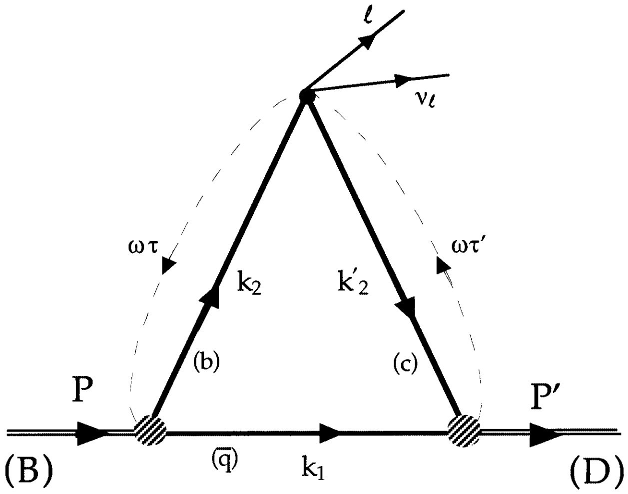

Fig. 4. Leading order contribution to the semi-leptonic B to D decay amplitude

we have

$$\begin{cases} \sqrt{M_B M_D} h_+ = \dfrac{1}{4(1+\alpha^2-2\alpha\eta)} \left[(1-\alpha)\left[(1-\alpha)\mathcal{X} \right.\right. \\ \qquad \left.\left. -(1+\alpha)\mathcal{Y}\right] + 2(1+\alpha)(1-\eta)\mathcal{Z}\right], \\ \sqrt{M_B M_D} h_- = \dfrac{1}{4(1+\alpha^2-2\alpha\eta)} \left[(1+\alpha)\left[-(1-\alpha)\mathcal{X} \right.\right. \\ \qquad \left.\left. +(1+\alpha)\mathcal{Y}\right] + 2(1-\alpha)(1+\eta)\mathcal{Z}\right]. \end{cases} \tag{32}$$

In leading order (spectator model), the amplitude is given by the diagram of Fig. 4. As already mentioned in Sect. 3, the spurion line connects two successive vertices in the light front "time" $\omega.x$. The matrix element of the current J_ρ is then easily evaluated using these rules. It is given by

$$\langle 0^-|J_\rho|0^-\rangle = \frac{1}{2}\int \mathrm{Tr}\left[\left(A_1' + A_2'\frac{\not\omega}{\omega\cdot p'}\right)\right.$$
$$\times(m_c + \not k_2')\gamma_\rho(m_b+\not k_2)\left(A_1 + A_2\frac{\not\omega}{\omega\cdot p}\right)\left.(m_q+\not k_1)\right]$$
$$\times\frac{1}{1-x}\frac{1}{1-x'}\frac{\mathrm{d}^3k_1}{(2\pi)^3 2\varepsilon_{k_1}}, \tag{33}$$

where x and x' are defined by

$$x = \frac{\omega\cdot k_1}{\omega\cdot p} \text{ and } x' = \frac{\omega\cdot k_1}{\omega\cdot p'}. \tag{34}$$

In (33), A_i (A_i') are the components of the initial (final) meson wave function.

4.2 Calculation of the form factors in the heavy quark limit

Using the matrix element of the current (33), and the definition of the form factors h_+, h_- from (32) in terms of $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ in (31), we can easily calculate these form factors in terms of the invariant scalar products of the various momenta involved in the process. We evaluate them in the B meson rest frame, in terms of the variable $\vec{k}$ defined in (5). We recall that in the infinite quark limit, the rest frame of the B meson is also the c.m. frame of the $Q\bar{q}$ system ($\vec{k}_1 + \vec{k}_2 = 0$). We shall also consider particular orientations of the light front position ω. For the calculation of electromagnetic form factors, we usually take $\omega\cdot q = 0$ [19], which corresponds to $q^+ = 0$ in the standard formulation of LFD. This choice cannot be done here because $Q^2 > 0$. As a convenient choice, which however keeps ω undetermined, we shall take $\vec{\omega}\cdot\vec{q} = 0$ in the rest frame of the B meson. With these conditions, it is easy to evaluate all the necessary scalar products in terms of $\mathbf{k}$ and η. With $p\cdot p' = p^0 p'^0 = M_B M_D \eta$ we have

$$p'^0 = \frac{M_B M_D \eta}{p^0} = M_D\eta \quad \text{and} \quad \frac{\omega\cdot p'}{\omega\cdot p} = \frac{p'^0}{p^0} = \frac{M_D}{M_B}\eta, \tag{35}$$

so that in the infinite mass limit we have $\alpha \equiv \eta$. As already seen in Sect. 3, the momentum fraction carried by the light quark, given by x as defined in (34), tends to zero as $1/M_B$ in the infinite mass limit, and similarly for x'. The parameters τ and τ', as indicated in Fig. 4, are also zero in this limit, as already seen in Sect. 3. In the B rest frame, we have $\vec{U} = 0, U^0 = 1$ and $U'^0 = \eta, |\vec{U}'| = (\eta^2-1)^{1/2}$. We have therefore $U\cdot k_1 = \varepsilon_k$ and $U'\cdot k_1 = \varepsilon_{k'} = \eta\varepsilon_k - (\eta^2 - 1)^{1/2}|\vec{k}|cos(\phi)$, where ϕ is the angle between $\vec{k}$ and $\vec{U}'$ or $\vec{p}'$. This latter relation enables us to express $\vec{k}'^2 = \varepsilon_{k'}'^2 - m_q^2$ in terms of η and the integration variable $\vec{k}$.

In the approximation where we neglect the relativistic component A_2 of the meson wave function (exact approximation in the infinite quark limit, see our discussion in Sect. 3), we finally get

$$\begin{cases} h_+ = \sqrt{M_B M_D}\displaystyle\int A_1 A_1' \left[\varepsilon_k + m_q - \sqrt{\frac{\eta-1}{\eta+1}}|\vec{k}|\cos(\phi)\right] \\ \qquad \times\dfrac{\mathrm{d}^3\vec{k}}{(2\pi)^3\varepsilon_k}, \\ h_- = 0. \end{cases} \tag{36}$$

As we already mentioned, we find that h_- is exactly zero, as it should. In the heavy quark limit, we have the relation (21) between the component A_1 and the non-relativistic wave function $\phi_{\mathrm{NR}}(\vec{k}^2)$ (when g_1 is identified with ϕ_{NR}), and similarly for A_1' in terms of $\phi_{\mathrm{NR}}(\vec{k'}^2)$. Moreover, the Luke theorem is nicely satisfied. Indeed, in the limit of zero recoil ($\eta = 1$), the h^+ form factor reads, with the components A_1 and A_1' given by (21),

$$h_+ = \sqrt{\frac{M_B}{m_b}\frac{M_D}{m_c}}\int g_1^2 \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3 2\varepsilon_k}. \tag{37}$$

In the heavy quark limit, $M_B/m_b \to 1$ and $M_D/m_c \to 1$, so that, with the normalization condition (22), the h^+

form factor is equal to 1 in the zero recoil limit. The analytic expression we then find for h_+ is identical to the one obtained in [6] using the Bakamjian–Thomas construction. Again relativistic corrections can be large since they involve ε_k and the light quark or antiquark mass.

5 Concluding remarks

Contrary to the old, and naive, belief that processes involving heavy quarks like c or b can be treated in a non-relativistic framework, it is now well accepted that a relativistic treatment of the decay and transition amplitude of heavy mesons is necessary.

We have shown in this work that a very convenient relativistic approach is provided by the covariant formulation of light-front dynamics. The formulation of the bound state wave function on the light front is necessary in order to give a physical sense to the constituent quark model which describes the mesons in terms of a two-body (quark–antiquark) bound state. The explicit covariance of our approach is then a powerful tool to make the connection between the relativistic two-body wave function and the non-relativistic Schrödinger wave function trivial.

As a starting point, we have investigated in the present work the leading $1/m$ contribution to the leptonic decay constant of B mesons, and the semi-leptonic B to D transition form factor. In this limit, we analytically recover the results obtained in the Bakamjian–Thomas construction in equal time dynamics, in a very transparent way.

In leading $1/m$ order, we recover also results obtained in the standard formulation of LFD [10–12, **?**,14]. This is a check that, in this limit, the calculations of LFD, as well as in the Bakamjan–Thomas construction in the equal time formalism, are exact. Therefore, no non-physical contributions arising from the lack of rotational invariance in the standard formulation of LFD can arise. This is however not the case if the $1/m$ corrections are included. Our covariant formulation of LFD is a very convenient, and systematic, way to have control over these non-physical contributions.

Corrections to the infinite mass limit can arise from several sources. First of all, they can come from relativistic corrections to the kinematics. These corrections are trivial to incorporate in a relativistic framework. The second type of corrections involves relativistic corrections to the two-body bound state wave function. As we have seen, the relativistic wave function of a pseudo-scalar meson has two components. Our formalism enables us to take into account the contributions from these two components, as given in (5) for instance. This is not possible in the Bakamjian–Thomas construction which starts from a single non-relativistic wave function.

The most difficult contribution to evaluate will certainly be the dynamics which generate both the two-body wave function but also the electroweak operator. At that level, one would have to assume a model to estimate these corrections. The one-gluon exchange is probably the most suited. It has already been applied in CLFD to estimate the two components of the pion wave function in the asymptotical region [19]. It can be applied as well for the processes under consideration in this work.

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